SHEAR RESISTANCE OF REINFORCED CONCRETE CORBELS FOR SHEAR KEYS

Zhe Qu$^{1)}$, Yusuke Maida$^{2)}$, Shoichi Kishiki$^{3)}$, and Hiroyasu Sakata$^{4)}$

1) Postdoctoral research fellow, Center for Urban Earthquake Engineering, Tokyo Institute of Technology, Japan
2) Graduate student, Dept. of Environmental Science and Technology, Tokyo Institute of Technology, Japan
3) Assistant professor, Structural Engineering Research Center, Tokyo Institute of Technology, Japan
4) Professor, Structural Engineering Research Center, Tokyo Institute of Technology, Japan

m.quzhe@gmail.com, maida.y.aa@m.titech.ac.jp, kishiki@serc.titech.ac.jp, sakata.h.aa@m.titech.ac.jp

Abstract: Rectangular reinforce concrete corbels with very small aspect ratio and uniformly distributed stirrups have been used as shear keys to anchor the gusset plate of buckling restrained braces in reinforced concrete buildings. An experimental study on the shear resistance of such shear keys is reported in this paper. Two series of shear test of the reinforced concrete corbels were carried out. The observed failure is in a very brittle manner, especially for those with nominal shear span-to-depth ratio less than 0.1. A strut-and-tie model is proposed to estimate the shear strength of the corbels. It gives satisfying estimate for all the specimens in the current test program, except the one with insufficient anchor of the stirrups.

1. INTRODUCTION

A continuously braced frame system was introduced as an attempt to promote the application of buckling restrained braces in newly-built reinforced concrete buildings (Sakata et al, 2011; Qu et al, 2011). The proposed arrangement of braces and details of fastening the braces to the concrete is demonstrated in Figure 1. The buckling restrained braces in the adjacent stories share the same gusset plate, which is fastened to the concrete beam-column joint by prestressing bolts and is kept by a pair of reinforced concrete (RC) corbels that are projecting from the column surface. In such a manner, the prestressing bolts are mainly responsible to resist the horizontal force while the RC corbels the vertical one.

![Figure 1](image1.png)

Figure 1 Continuously braced frame with buckling restrained braces

The reinforcement shown in Figure 2 is generally adopted for such RC corbels, which work as shear keys in the proposed system. It consists of a set of uniformly distributed stirrups and several framing bar to anchor them. The stirrups extend into the concrete column to a certain length in order to introduce adequate bond to develop their strength.

These corbels are also featured by its extremely small aspect ratio for that the length of the corbel, $L$, in Figure 2, is usually in proportion to the size of the gusset plate. The length to height ratio, $L/H$, of these corbels, may be frequently less than 0.5. The current paper is to investigate the mode of failure of such RC corbels through experimental tests and to estimate its shear strength using strut-and-tie models.

![Figure 2](image2.png)

Figure 2 RC corbel for shear key

2. EXPERIMENTAL PROGRAM

Two series of test, namely Series A and Series B, of RC corbel specimens with different details, were carried out.

2.1 Specimens

Specimens in either Series A or Series B have identical dimensions, as depicted in Figure 3. Series A specimens
have a aspect ratio $L/H$ of 0.3, which leads to a nominal shear span-to-depth ratio as small as 0.15, assuming uniform pressure on the corbel edge. This ratio for Series B is even smaller, about 0.09.

Major properties, including the concrete compressive strength, $f'_{c}$, stirrup yield strength $f_y$, stirrup nominal diameter, $d_s$, total number of stirrup legs, $n_{leg}$, and the ratio of anchorage length to nominal diameter of bar, $l_a/d_s$, of the specimens in Series A and B are listed in Table 1.

A1 is regarded as a control specimen in Series A. The other four specimens are designed to show the influence of the loading histories (A2), concrete strength (A3), amount of stirrups (A4) and anchorage of stirrups (A5) on the behavior of the corbel.

Specimens in Series B were actually taken from the tested specimens of subassemblies, each of which consists of an RC beam framing into an RC column with connections for buckling restrained braces (Qu et al, 2011). Some minor cracks were observed on the corbels during the subassembly test. However, the influence of this slight damage to those corbels on their ultimate strength was considered negligible. Each subassembly contains a pair of identical corbels, as can be seen in Figure 4(b). The beam in each subassembly has been cut off. A total of two such subassemblies were tested for their corbels, which give the four corbel specimens in Series B. Hence, B1a and B1b are identical and belong to the same subassembly, so are B2a and B2b. The only difference between B1 and B2 is the amount of stirrups in the corbels.

### 2.2 Loading setup and measurement

Series A specimens were subjected to cyclic loading by means of two actuators on both sides of the corbel, pushing it by turns, as demonstrated in Figure 4(a). Cyclic loading was conducted in both directions for all the specimens except Specimen A2, for which, the cyclic loading was only in a single direction. That is, to load to peak and unload to zero, and then reload in the same direction.

For Series B, a self-balanced loading system, as depicted in Figure 4(b), was adopted. The loading jig was connected to 4 high-strength steel rods, the other ends of which were connected with an H steel beam. An actuator was installed between the H steel beam, which was resting on a roller cushion, and the end of the RC subassembly, which was fixed to the ground, pushing the two apart from each other and thus pulling the loading jig to impose shear force on the corbel. Monotonic loading up to the failure of the corbel was applied to Series B specimens.

The shear deformation of each corbel in Series A was recorded through six displacement transducers installed between half span of the corbel and the top surface of the underneath concrete monolith. Their locations are given in Figure 3(a). The average of the displacement of the two transducers closest to the loading edge was taken as the shear deformation of the corbel under that load.

Two displacement transducers were used to measure the slip of each corbel in Series B. They were installed between the corner of the loading jig and the underneath concrete column, as can be seen in Figure 3(b). For both series, the strain of the corbel stirrups was recorded through strain gages located at the shear surfaces, i.e., the bottom of the corbels. A pair of strain gages was adopted for a single leg of stirrups and their average reading is taken as the strain of the stirrup leg.
3 EXPERIMENT RESULT

3.1 Series A

Shear force-displacement curves of Series A specimens are depicted in Figure 5. The curves of all the other specimens are compared with that of A1 in these graphs to show the influence of various parameters. The loading path seems to have a surprisingly significant influence on the maximum strength of the corbel (Figure 5(a)). The maximum strength is reduced by about 25% in the one-way cyclic test (A2) compared with that in the standard test (A1). Reason for this reduction is still not understood. For A3 and A4, it is observed that, as expected, the shear strength of the corbels decreases with the reduction in the concrete strength or the amount of stirrups (Figure 5(b) and (c)). Specimen A5 seems to have experienced premature failure due to insufficient anchorage of the corbel stirrups. Its shear strength is only about 43% of that of A1 (Figure 5(d)).

The shear strength of the tested corbels is listed in Table 2 near the end of this paper.

3.2 Series B

The load-displacement curves of corbels in Series B are depicted in Figure 6.

For all the specimens, the load increases almost linearly with the displacement up to a very high level of shear force. It is then followed by a sudden brittle failure, which is accompanied by the breaking up of the concrete cover. The corbels with half amount of stirrups (B2a and b) exhibit only about 25% decrease in shear strength as compared with B1a and b.

Figure 7 depicts the failure surface of the specimens, above which the concrete cover was destroyed. Before this, several minor diagonal cracks also developed primarily below the failure surface. These cracks, as well as the ultimate failure surface, indicated approximately the direction of principle strain in the concrete. Rotation of this direction close to the loading edge can be observed.

3.3 Strain distribution in stirrups

The strain in the stirrups at the maximum shear force is retrieved from the test database and is denoted as $\varepsilon_{max}$. It is then normalized by the yield strain $\varepsilon_y$ of the stirrups and is depicted in Figure 8. The horizontal axis is the distance to the loading edge normalized by the height, $H$, of the corbel. It is observed that $\varepsilon_{max}$ generally reduces linearly with the distance of the stirrup to the loading edge. Several obvious exceptions should be noted.

(1) The strain in the stirrups of Specimen A5 seems to have very insufficiently developed, even in those closest to the loading edge. It is likely that the stirrups experienced premature anchorage failure due to the insufficient development length.

(2) For Series B, the strain in the stirrups closest to the loading edge seems to have increased wildly and is much greater than that in the following stirrups.

(3) The $\varepsilon_{max}$ to $\varepsilon_y$ ratios of B1a and B1b exhibit a zigzag
distribution along the distance. This is considered the consequence of un-uniformly distributed deformation along the width of the corbel. It was observed, after the crushed concrete cover was removed, that the top horizontal portions of the stirrups, especially those close to the loading edge, were obviously bent with greater deformation in the middle and smaller one at both ends. Figure 9 gives an example of Specimen B1a. The gages of the stirrups were always attached to the vertical portion of a stirrup, i.e., the farthest end from the center. The stirrups of B1a and B1b were such arranged that an inner stirrup follow an outer stirrup, as can be seen in Figure 9. This may occasion that the strain in the inner stirrup becomes greater than that in the outer one, which was a little bit closer to the loading edge than it is.

4. STRUT-AND-TIE MODEL

A strut-and-tie model, as shown in Figure 10, is adopted to predict the shear strength of the corbels. It is a modified version of the strut-and-tie model used by Russo et al (2005) in their derivation. The following two major modifications are made.

(1) By assuming that the shear contribution of the strut-and-tie system, which is formed by a concrete strut and the primary stirrups close to the loading plate, and that of the secondary stirrups are independent, Russo et al (2005) proposed a two-term formula to predict the shear strength of the corbel. The first term is for the strut-and-tie system and the second for the secondary stirrups. However, instead of evaluating the dowel action of the secondary stirrups, they again assumed a strut-and-tie system consisting of the secondary stirrups when evaluating the second term. In such a manner, they may have double counted the strength of the concrete strut. In this paper, the second term of the formula by Russo et al (2005) is removed and the imperial factor of 0.8 on the first term by data fitting is correspondingly abandoned. Thus, Eq. 1 is used herein to evaluate the shear strength, $V_{uc}$, of RC corbels.

$$V_u = \sigma_d b l \cos \theta$$

where $\sigma_d$ is the softened compressive strength of the diagonal concrete strut; $b$ is the width of the corbel; $l$ and $\theta$ is the depth and inclination angle of the concrete strut, respectively.

All these variables can be explicitly evaluated by Eq. 2-4 proposed by Russo et al (2005).

$$\sigma_d = \begin{cases} 0.9 f'_{ce} & \text{for } f'_{ce} < 42 \text{MPa} \\ \sqrt{\frac{5.8 f'_{ce}}{1 + 400 f'_{ce}}} & \text{for } f'_{ce} \geq 42 \text{MPa} \end{cases}$$
\[ l = kd = \left( \sqrt{(n\rho_s)}^2 + 2n\rho_s - n\rho_s \right) \cdot d \quad (3) \]

\[ \theta = 2 \arctan \left( \frac{1 + \left( \frac{a^2}{d^2} \right) - \left( 1 - \frac{k^2}{4} \right)}{\frac{a - k}{2}} \right) \quad (4) \]

where \( n = E_s/E_c \) is the ratio of steel and concrete Young’s modulus; \( f_c \) is the tensile strength of concrete; \( d \) is the effective depth of the corbel and \( \rho_s \) is the primary stirrups ratio of the corbel.

(2) Because the stirrups are uniformly distributed along the depth of the corbels, no primary stirrups can be readily extinguished from others. As a result, effective \( d \) and \( \rho_s \) are proposed herein. A linear distribution of stirrup stress is assumed. The stirrup stress on the loading edge is assumed to be the yield strength, \( \sigma_y \), while that on the other end is 0 (Figure 10). Compared with the measured strain of the stirrups at failure in Figure 8, the assumed stress distribution is reasonable and conservative.

With the assumed stirrup stress distribution, the effective depth of the tie and the effective stirrup ratio can then be calculated by Eq. 5 and 6.

\[ d = \frac{2}{3} h \quad (5) \]

\[ \rho_s = 0.5 \frac{n_{eg} A_{st}}{bh} \quad (6) \]

where \( A_{st} \) is the cross-section area of a single stirrup leg.

The shear strengths calculated by Eq. 1-4, denoted as \( V_{u,cal} \) and those obtained in the test are compared in Table 2 and Figure 11. The calculated results generally agree well with the experiment and fall into a 20% error band. An exception is specimen A5, whose stirrups sustained premature bond-slip failure due to insufficient anchorage.

Table 2 Test and predicted shear strength

<table>
<thead>
<tr>
<th>ID</th>
<th>( V_{u,test} ) (kN)</th>
<th>( V_{u,cal} ) (kN)</th>
<th>( V_{u,test} / V_{u,cal} )</th>
</tr>
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<tbody>
<tr>
<td>A1</td>
<td>1713.1</td>
<td>1521.5</td>
<td>1.13</td>
</tr>
<tr>
<td>A2</td>
<td>1281.4</td>
<td>1521.5</td>
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<td>A3</td>
<td>1396.8</td>
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<td>A4</td>
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<tr>
<td>A5</td>
<td>730.5</td>
<td>1521.5</td>
<td>0.48</td>
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<td>B1a</td>
<td>1176.3</td>
<td>1053.4</td>
<td>1.12</td>
</tr>
<tr>
<td>B1b</td>
<td>1145.6</td>
<td>1053.4</td>
<td>1.09</td>
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<tr>
<td>B2a</td>
<td>796.4</td>
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<tr>
<td>B2b</td>
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<td>780.3</td>
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</table>

5. CONCLUSIONS

The experiment program of evaluating the shear resistance of a type of very deep RC corbels is described. These corbels may fail in a very brittle manner. This suggests that adequate safety margin of shear strength should be guaranteed in the design of it as shear keys.

Based on an assumption of stress distribution in the stirrups that is supported by the test data, a strut-and-tie model, which is a modified version of a previous model, is proposed and proved to give satisfying estimate of the shear strength of the specimens in the current test.

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