Stiffness demand for the outer casing in a buckling restrained brace

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A simple model shown in Figure 1 is used to derive the required stiffness of the casing of buckling restrained braces (BRB).

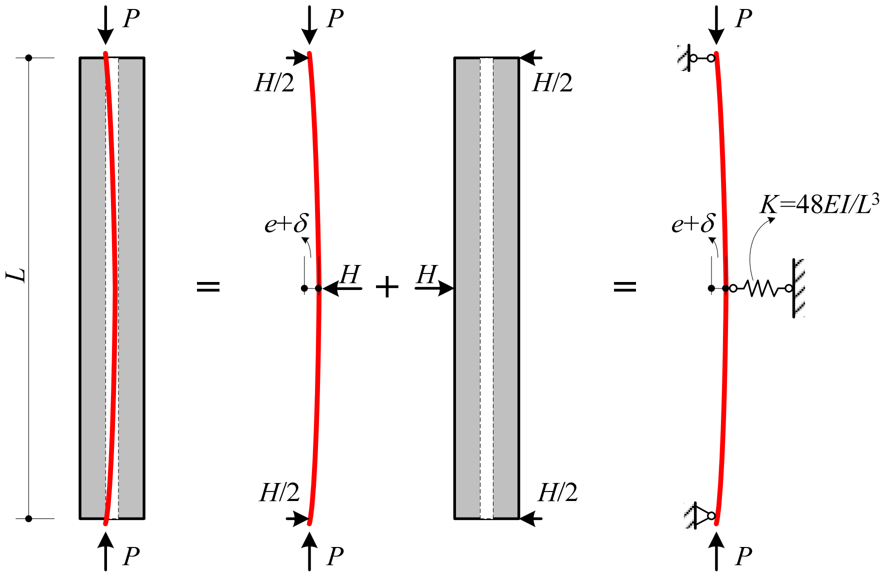

Figure 1. A simple to derive the required stiffness of the casing in a BRB

From the equivalent of moment around the middle point of the core in Figure 1(b), it must have:

$$P(e+\delta) = HL/4 + M_{\text{core}} \approx HL/4 \tag{1}$$

where e is the initial imperfection of the core and δ is the deformation induced by the external compression P; L is the length of the brace and H denotes the horizontal constraining force provided by the casing; M_{core} is the moment at the middle span section of the steel core, and could be reasonably assumed to be zero, especially when the core has yielded in compression.

On the other hand, the deflection of the casing at the middle of its span would be

$$\delta = H/K = HL^3/(48EI) \tag{2}$$

where K=48EI/L^3 is the bending stiffness of the casing, E and I can be conservatively taken as the elastic modulus and the moment of inertia of only the steel tube.

The horizontal restraining force H can then be worked out by equating the deformation of the core and the casing induced by the compression P:

$$H = \frac{e}{\dfrac{L}{4P} - \dfrac{L^3}{48EI}} \tag{3}$$

In order to provide effective restrain to the core from lateral buckling, H should always stay positive. Thus the denominator of the above equation should always stay positive, i.e.:

$$\frac{L}{4P} - \frac{L^3}{48EI} > 0 \qquad (4)$$

A criterion of determining the stiffness of the casing can then be obtained by putting the above equation in the following form:

$$\frac{12EI}{L^2} > P_y \qquad (5)$$

Here we assume that the maximum compressive force that the core can ever sustain is its yield strength P_y. This equation excludes the influence of the degree of imperfection. Instead, another criterion may be drawn by assuming that the moment in the middle of the tube would not exceed its yield moment, M_y, i.e.,

$$M = \frac{HL}{4} = \frac{e}{\frac{1}{P} - \frac{L^2}{12EI}} < M_y = \frac{\sigma_y I}{\frac{D}{2}} \qquad (6)$$

where D and σ_y is the diameter and the yield strength of the steel tube, respectively.

Rearranging Eq. 6 and substituting P by P_y yields

$$\frac{12EI}{L^2} > P_y\left(1 + 12 \cdot \frac{E}{2\sigma_y} \cdot \frac{D}{L} \cdot \frac{e}{L}\right) \qquad (7)$$

It is similar to Equation 5 but it takes into account the influence of imperfection. With large imperfection, the tube needs to be stronger or stiffer to prevent the development of the global buckling of the core.

On the other hand, it is of the same form as the criterion proposed by Wada and Nakashima (2004) (Eq. 8).

$$P_e = \frac{\pi^2 EI}{L^2} > P_y\left(1 + \pi^2 \cdot \frac{E}{2\sigma_y} \cdot \frac{D}{L} \cdot \frac{e}{L}\right) \qquad (8)$$

Eq. 7 and Eq. 8 share the same form but the coefficient on both sides is 12 in Eq. 7 and is π^2 in Eq. 8. This is the consequence of different functions of lateral deformation pattern that are adopted in the derivation. It was assumed in the derivation of Eq. 8 that the lateral deformation of the tube and that of the core follows the same sine function. By contrast, it is assumed for Eq. 7 that the tube is carrying a concentrated load at its middle point and its deflection curve is the result of this concentrated load.

Reference:

Wada, A.; Nakashima, M. (2004). From infancy to maturity of buckling restrained braces research. Proc. 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, Paper No. 1732.

Reminder: How to get Eq.7 from Eq. 6:

$$\frac{e}{\frac{1}{P} - \frac{L^2}{12EI}} < \frac{\sigma_y I}{\frac{D}{2}} \Rightarrow \quad \frac{e}{\frac{12EI}{PL^2} - 1} < \frac{\sigma_y I}{\frac{6DEI}{L^2}} \Rightarrow \quad \frac{12EI}{PL^2} - 1 > \frac{6DEe}{\sigma_y L^2} \Rightarrow \quad \frac{12EI}{L^2} > P\left(1 + \frac{6DEe}{\sigma_y L^2}\right) \Rightarrow$$

$$\frac{12EI}{L^2} > P_y\left(1 + \frac{6E}{\sigma_y} \cdot \frac{D}{L} \cdot \frac{e}{L}\right)$$

Thank Igor Marinovic from BlueScope Buildings North America Inc. for checking the derivation and pointing out a mistake in Equation 6.